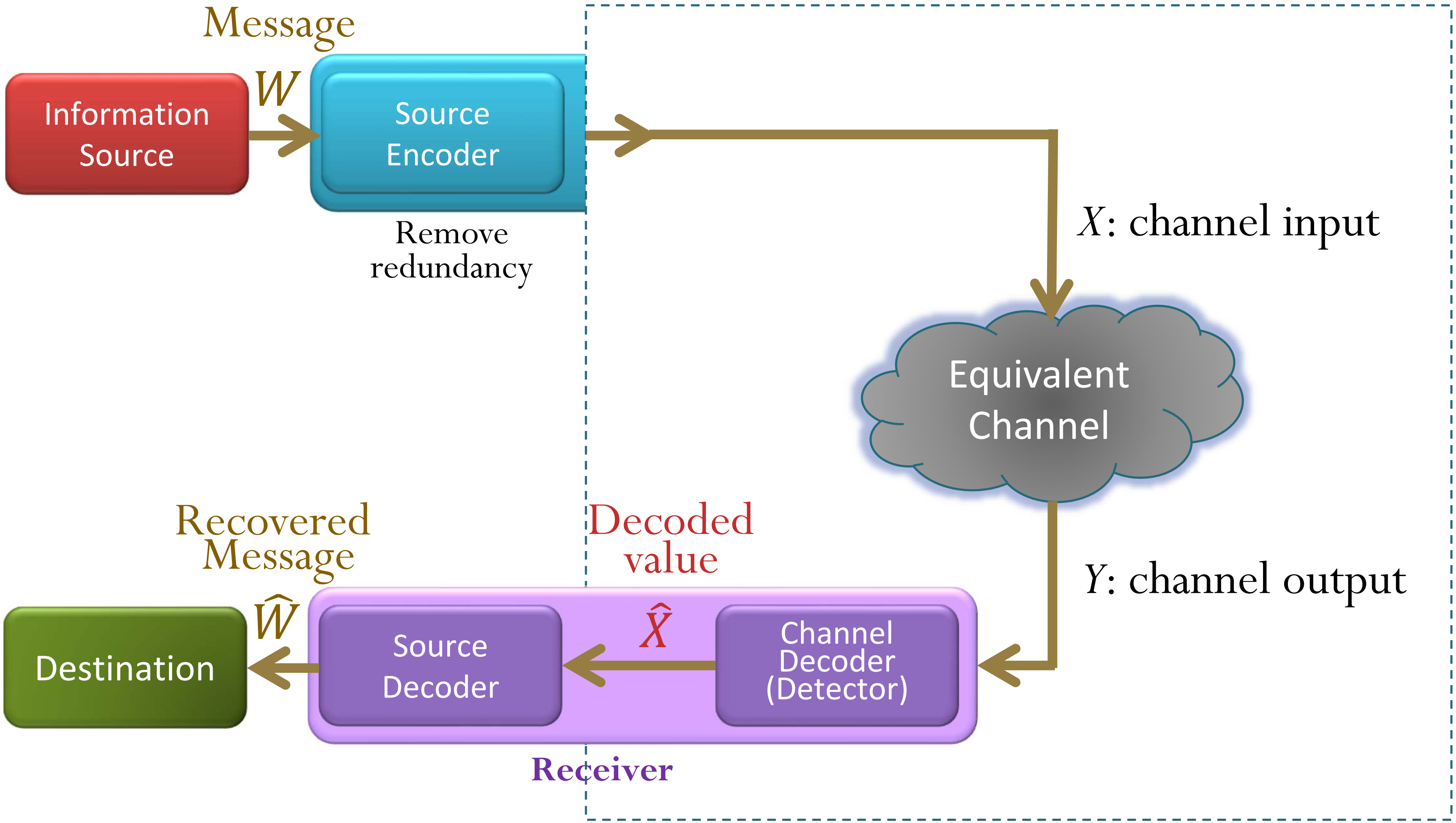
Digital Communication Systems
EES 452

Asst. Prof. Dr. Prapun Suksompong
prapun@siit.tu.ac.th

3 An Introduction to Digital Communication Systems Over Discrete Memoryless Channel

3.4 Optimal Block Decoding for Communications Over BSC

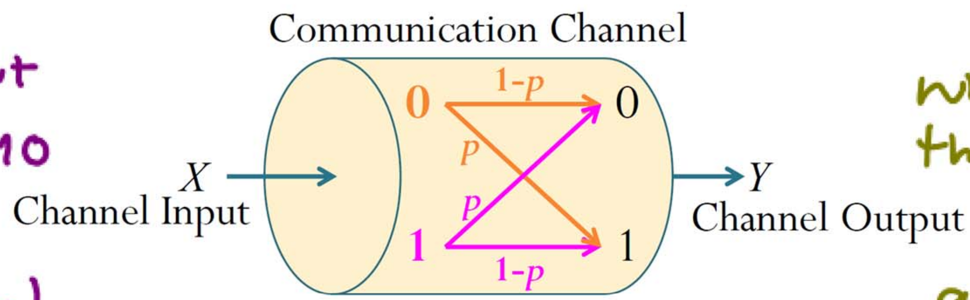
System Model for Section 3.2-3.3



Review: EES315 (2020)

Example 6.58. ***Digital communication over unreliable channels***: Consider a digital communication system through the **binary symmetric channel** (BSC) discussed in Example 6.18. We repeat its compact description here.

Suppose you put 010110 into this channel.



What is the probability that you get 011010 as output?

010110

↓↓↓↓↓↓

011010

90

$$(1-p)(1-p)pp(1-p)(1-p) = (1-p)^4 p^2$$

[http://www2.siit.tu.ac.th/prapun/ees315/EES315%206.3%20u2.pdf]

Review: EES315 (2020)

EES 315: In-Class Exercise # 13

1) [Digital Communications] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.4. Assume bit errors occur independently. Your answers for parts (a) and (b) should be of the form X.XXXX.

$p = 0.4$

a) Suppose we input bit sequence "1010101" into this channel.

The BSC is used $n = 7$ times (this is the number of bits in both the input and the output sequences.

1010101
↓↓↓↓↓↓↓

i) What is the probability that the output is "1000001"?

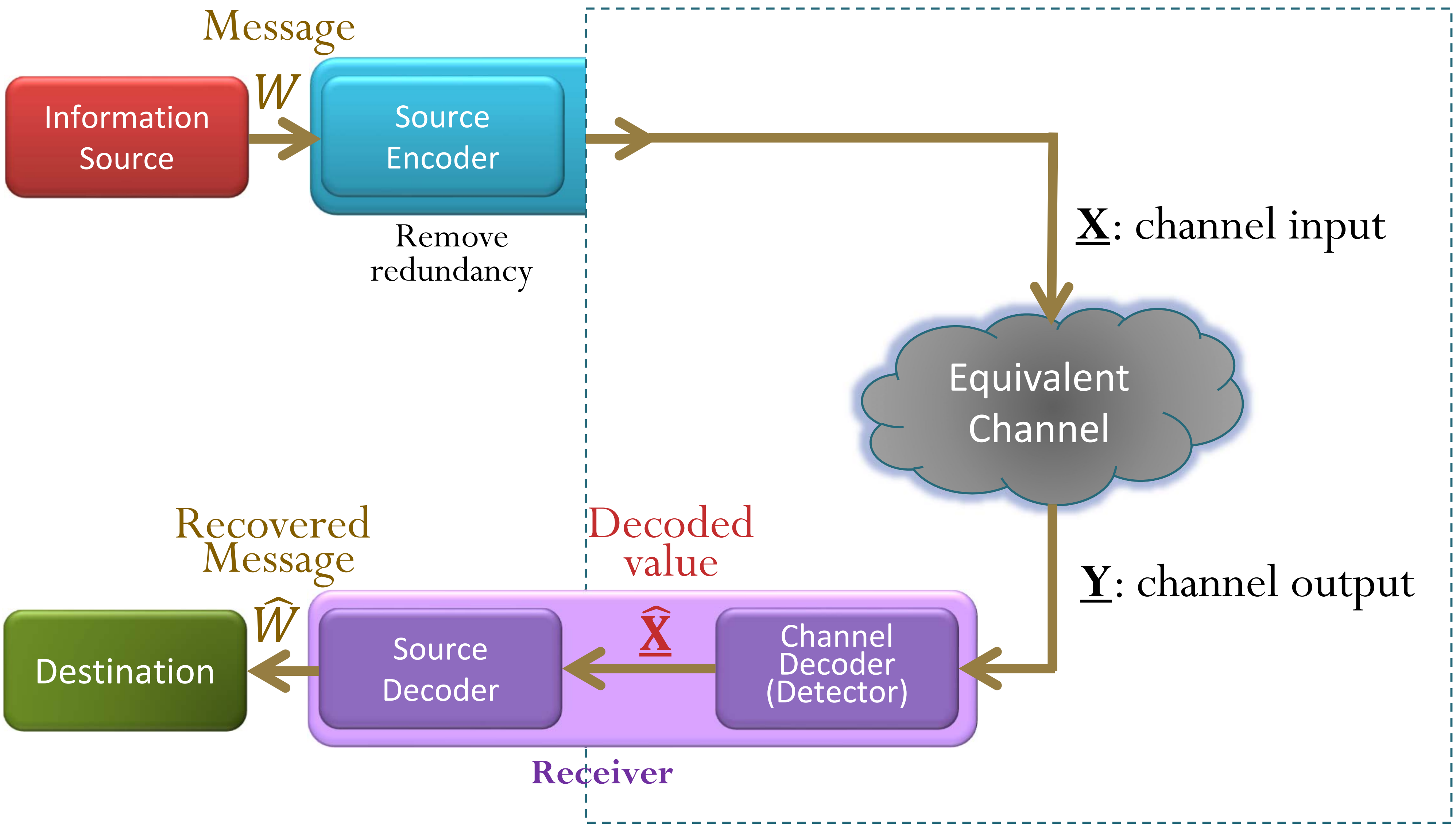
This is similar to Example 6.54 in the lecture notes. However, note that p here is the probability of bit error. So, "success" here corresponds to the case that we have bit error; and "failure" corresponds to the case that the bit is unchanged when travels across the channel.

$$(1-p)\times(1-p)\times p\times(1-p)\times p\times(1-p)\times(1-p) = (1-p)^5\times p^2 = 0.6^5\times 0.4^2$$

$$= \frac{972}{78125} \approx 0.0124$$

[http://www2.siit.tu.ac.th/prapun/ees315/EES315_Exercise_2020_13_Sol.pdf]

System Model for Section 3.4



Channel Decoder

[3.57]

$$\hat{\underline{\mathbf{x}}}_{\text{MAP}}(\underline{\mathbf{y}}) = \hat{\underline{\mathbf{x}}}_{\text{optimal}}(\underline{\mathbf{y}})$$
$$= \arg\max_{\underline{\mathbf{x}}} P\left[\underline{\mathbf{X}} = \underline{\mathbf{x}} \middle| \underline{\mathbf{Y}} = \underline{\mathbf{y}}\right]$$
$$= \arg\max_{\underline{\mathbf{x}}} Q(\underline{\mathbf{y}}|\underline{\mathbf{x}})\, p(\underline{\mathbf{x}})$$

[Ex 3.54]

BSC

$$Q(\underline{\mathbf{y}}|\underline{\mathbf{x}}) = p^{d(\underline{\mathbf{x}},\underline{\mathbf{y}})}(1-p)^{n-d(\underline{\mathbf{x}},\underline{\mathbf{y}})}$$

ML decoder is the same as the MAP decoder when the codewords are equally likely.

$$\hat{\underline{\mathbf{x}}}_{\text{ML}}(\underline{\mathbf{y}}) = \arg\max_{\underline{\mathbf{x}}} P\left[\underline{\mathbf{Y}} = \underline{\mathbf{y}} \middle| \underline{\mathbf{X}} = \underline{\mathbf{x}}\right]$$
$$= \arg\max_{\underline{\mathbf{x}}} Q(\underline{\mathbf{y}}|\underline{\mathbf{x}})$$

$$\hat{\underline{\mathbf{x}}}_{\min d}(\underline{\mathbf{y}}) = \arg\min_{\underline{\mathbf{x}}} d(\underline{\mathbf{x}},\underline{\mathbf{y}})$$

[Ex 3.56]

For BSC with $p < 0.5$, ML decoder is the same as the min distance decoder

[3.57]

Some results from Section 3.3-3.4

- **MAP decoder** is optimal. (It minimizes $P(\mathcal{E})$).
- **ML decoder** is suboptimal.
  - However, in many cases, it can be optimal (same $P(\mathcal{E})$ as the MAP decoder),
    - for example, when the codewords are equally-likely.
- ML decoder is the same as the **minimum distance decoder** when the crossover probability of the BSC p is <0.5 (which is usually the case).